Mathematics in Preschool

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Objectives

1. Review Developmentally Appropriate Practice (DAP)
2. Examine recent research on early numeracy development
3. Introduce early numeracy concept trajectories
4. Illustrate concept trajectories with video examples
5. Identify evidence-based instructional practices to support early numeracy development of all young children, ages three to five
6. Discuss video and case study examples of early math activities with young children who have disabilities

Why numeracy skills?

- Want to prepare preschoolers for kindergarten and beyond
- The numeracy skills children know when they enter kindergarten and first grade predicts their mathematic achievement for years to come

Developmentally Appropriate Practice (DAP)

National Association for the Education of Young Children (NAEYC)

Key messages

- All teaching practices should be appropriate to children’s age and developmental status
- Reduce the achievement gap
- Comprehensive, effective curriculum
- Improving teaching and learning

Copple & Bredekamp (2009)
What does the research say?


- This meta-analysis showed the strongest predictors of later achievement are school-entry math, reading and attention skills
- Early math skills have the greatest predictive power of later academic achievement
- By contrast, measures of socio-economic behaviors were generally insignificant predictors of later academic performance

What does the research say?


- The committee found that although virtually all young children have the capability to learn and become competent in math, most of the potential to learn math in the early years is not currently realized
- This stems from lack of opportunity to learn math in early childhood settings and at home

What does the research say?


- Cognitive research shows that young children are capable of learning more and deeper mathematics than usually assumed
- Early childhood teachers have not been prepared to teach math, may be afraid to teach math, or feel that it is not important to teach math
What does the research say?

- Two pictures hang in front of a six-month-old child. The first shows two dots, and the other has three dots. The infant hears three drumbeats. Her eyes move to the picture with three dots.
- Research shows this is one of the abilities very young children develop.
  - Why would this skill be important for young children?

Adapted from the National Research Council (2009) Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity

What do national organizations say?


- Prior to kindergarten, children have the interest and capacity to acquire math concepts through experiences and play
- Many early childhood programs do not extend children’s math knowledge

The preschooler’s brain

- Natural curiosity about everything
- Innate number sense
- Brains develop most when challenged with complex, novel activities and not rote learning
- Preschoolers do not perceive situations, problems or solutions the same way adults do
- Young children see the world through an integrated lens, not as separate subject areas

Adapted from Sousa (2008) How the Brain Learns Mathematics
### Typical time spent on numeracy

- National Center for Early Development and Learning (NCEDL) in two studies of state-operated facilities documented the following findings:
  - Children were exposed to math 6% of the time
  - Math instruction occurred during whole group activities
  - Teachers focused on student performance of discrete skills
  - Children were less often exposed to instruction that was conversational, interactive, focused on understanding or problem solving

  *Adapted from the National Research Council (2009) Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*

### What is high quality math in early childhood?

- Does **not** involve pushing early elementary arithmetic onto younger children
- Allows children to experience mathematics as they play in and explore the world around them

### Observing the child and asking *guiding questions* to help them build their knowledge along a trajectory

- "How many strawberries do you have on your plate?"
- "Do you need more cotton balls?...How many?"
- "How could you make your tower taller?"
- "Are you sure you have five? Count with me..."
Reflection

Take a moment to reflect on strategies used in your preschool setting.

- How do you currently incorporate guiding questions related to math concepts into the classroom routine? During which activities?
- What new types of guiding questions related to math concepts could be incorporated into the classroom routine?
- How do I model language related to math concepts for classroom staff? In which classroom routines?

Learning Trajectories

- Based on research on the natural developmental of a number concept
- Incorporate a big idea in math, provide a path for learning, and instructional tasks that match each part of that path for learning
- Follow the mathematic skill level of children in their way of thinking

Adapted from Clements and Sarama (2009) Learning and Teaching Early Math: The Learning Trajectories Approach

Early numeracy skills

Three Key Areas

- Number and Operation*
- Shape
- Measurement

*Most instructional time should be spent on number
What must a child know in order to be able to ‘count’ correctly?

- Two types of counting
  - If we ask children to count, they may just start rote counting “one, two, three...” when we want them to tell us “how many”
  - Understanding the difference takes time and practice

Concepts in counting:

- Recognize counting words
- The sequence of numbers
- One-to-one correspondence
- Cardinality
- Reverse of cardinality

One-to-one correspondence: assigning one and only one number to each object in a group

- Children have to know the sequence of numbers, remember the sequence, and know where they are in the sequence as they count in order to master one-to-one correspondence
Number and operation

The sequence of numbers:
- 1-10
- 11-13, 15
- 14, 16-19
- 20-29
- 30-39
- and so on...

Number and operation

Cardinality
- Child moves from just saying the number sequence to understanding that the number stated answers the question “how many?”
- Example: Susie just counted six raisins, appropriately touching each raisin while saying the number aloud. When her teacher asks Susie how many raisins she has, Susie answers "six" without recounting the raisins.

Number and operation

Reversal of cardinality
- Child can be asked to “give me eight blocks” and is able to count out the correct amount.
- Why would this be more difficult?
- Example: When the child gives eight blocks, do they count each block as they give it?
### Counting Trajectory

<table>
<thead>
<tr>
<th>Age</th>
<th>Developmental Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>late 1</td>
<td>Chants “sing-song” or sometimes indistinguishable number words</td>
</tr>
<tr>
<td>2</td>
<td>Verbally counts with number words, not necessarily in correct order above “five”</td>
</tr>
<tr>
<td></td>
<td>If child knows more number words than number of objects, they rattle them off quickly at end. If more objects, “recycles” number words</td>
</tr>
<tr>
<td>3</td>
<td>Verbally counts to ten with some correspondence</td>
</tr>
<tr>
<td>Later 3</td>
<td>Keeps one-to-one correspondence between counting words and objects for at least small groups. May recount if asked a second time, “How many?”</td>
</tr>
<tr>
<td>4</td>
<td>Accurately counts objects in a line to 5 and answers the “How many?” question with the last number counted</td>
</tr>
</tbody>
</table>

Adapted from Clements & Sarama (2009) *Learning and Teaching Early Math: The Learning Trajectories Approach*

### Counting Trajectory

<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Later 4</td>
<td>Counts arrangements of objects to 10. May be able to tell the number just after or just before another number, but only by counting from 1</td>
</tr>
<tr>
<td></td>
<td>Counts out objects to 5, with one-to-one-correspondence</td>
</tr>
<tr>
<td>5</td>
<td>Counts out objects to 10 and beyond, keeps track of objects that have or haven’t been counted</td>
</tr>
<tr>
<td></td>
<td>Gives next number (usually to 20 or 30), separates decade and one parts of a number</td>
</tr>
<tr>
<td></td>
<td>Recognizes errors in other’s counting</td>
</tr>
</tbody>
</table>

Adapted from Clements & Sarama (2009) *Learning and Teaching Early Math: The Learning Trajectories Approach*

### Killian

- Killian at grandma’s house
  - Turned 4 years old this month
  - Is not enrolled in any type of special education program
- Dots on a plate
- Dinosaurs in an egg carton
Number and operation

Components of Operation

- Subitizing
- Comparison words
- Modeling

What did you see?
Number and operation

**Subitizing**: knowing how many objects are in a collection without counting

- What is its role in operation?

Number and operation

**Quantity comparison**

Number and operation

**Comparison words**
- Bigger, smaller
- Longer, shorter
- Less, more
- Lighter, heavier

- How are these words and concepts included in the classroom routine?
Comparison Trajectory

<table>
<thead>
<tr>
<th>Age</th>
<th>Developmental Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Puts objects, words, actions in one-to-one or one-to-many correspondence</td>
</tr>
<tr>
<td>2</td>
<td>Implicitly sensitive to the relation of “more than/less than” involving very small numbers</td>
</tr>
<tr>
<td></td>
<td>Compares collections that are quite different in size. If same size, numbers must be small (one or two)</td>
</tr>
<tr>
<td>3</td>
<td>Compares collections of 1-4 items verbally or nonverbally. Has to be same item</td>
</tr>
</tbody>
</table>

Adapted from Clements & Sarama (2009) Learning and Teaching Early Math: The Learning Trajectories Approach

Comparison Trajectory

<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Compares groups of 1-6 by matching, not the same objects.</td>
</tr>
<tr>
<td></td>
<td>Accurately counts two equal groups, but the more spread out, or larger, will be more</td>
</tr>
<tr>
<td>5</td>
<td>Compares with counting, even when larger collection’s objects are smaller; later, figures out how many more or less</td>
</tr>
<tr>
<td></td>
<td>Names a small number for sets that cover little space and a big number for sets that are spread out</td>
</tr>
</tbody>
</table>

Adapted from Learning and Teaching Early Math: The Learning Trajectories Approach, Clements & Sarama, 2009

Comparison example

- Two plates of macaroni
Number and operation

Modeling: To model or act out change situations that are said in words and shown with actions

- How do you model number and operation in your setting?

Killian

Modeling one-to-one correspondance

- Dinosaurs and eggs

Operation Trajectory

<table>
<thead>
<tr>
<th>Age</th>
<th>Developmental Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sensitivity to adding and subtracting perceptually combined groups. No formal adding</td>
</tr>
<tr>
<td>2-3</td>
<td>Adds and subtracts very small collections nonverbally.</td>
</tr>
<tr>
<td>4</td>
<td>Finds sums for joining problems up to $3 + 2$ by counting with objects.</td>
</tr>
<tr>
<td>4-5</td>
<td>Finds sums for joining and part-part-whole by direct modeling, counting-all with objects. Solves take-away problems by separating with objects.</td>
</tr>
</tbody>
</table>

Adapted from Clements & Sarama (2009) Learning and Teaching Early Math: The Learning Trajectories Approach
Killian

Operation
- Counting pea pods

Shape

- Recognition of basic shapes
  - Fitting objects together, describing attributes of shapes

- Classification and sorting
  - Putting toys away in appropriate location, selecting items needed for a specific activity (napkin, spoon and cup for snack)
  - Example: find all of the...(red buttons, small spoons, large beans)

- Application to the world around us
  - Self-help skills (engaging zipper on coat, orienting clothing the proper direction), drawing/writing (tracing shapes or letters)
Shape Trajectory

<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>Compares real-world objects and says whether they are the same or different</td>
</tr>
<tr>
<td></td>
<td>Matches familiar shapes</td>
</tr>
<tr>
<td></td>
<td>Matches familiar shapes with different sizes</td>
</tr>
<tr>
<td>3</td>
<td>Recognizes and names squares and circles, less often triangles</td>
</tr>
<tr>
<td></td>
<td>May rotate shape to mentally match to a prototype</td>
</tr>
<tr>
<td>3-4</td>
<td>Matches a wider variety of shapes with same size and orientation</td>
</tr>
<tr>
<td></td>
<td>Matches a wider variety of shapes with different sizes and orientations</td>
</tr>
<tr>
<td>4</td>
<td>Recognizes some less typical triangles, some rectangles but not rhombuses</td>
</tr>
<tr>
<td></td>
<td>Says two shapes are the same after matching one side on each</td>
</tr>
<tr>
<td>4-5</td>
<td>Looks for differences in attributes, but may examine only part of a shape</td>
</tr>
<tr>
<td>5</td>
<td>Recognizes more objects, looks for differences in attributes</td>
</tr>
<tr>
<td></td>
<td>Names a variety of shapes and classifies by an attribute or two</td>
</tr>
</tbody>
</table>

Adapted from Clements & Sarama (2009) *Learning and Teaching Early Math: The Learning Trajectories Approach*

Killian

Shape

- Labeling triangles
**Measurement**

- Assigns a number to a measurable attribute of an object, usually length, weight or mass
- Young children can compare values grossly with appropriate words (big/little, more/less, a lot/a little)

**Measurement Trajectory**

<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Does not identify length as attribute</td>
</tr>
<tr>
<td>3</td>
<td>Identifies length/distance as attribute, may understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative</td>
</tr>
<tr>
<td>4</td>
<td>Physically aligns two objects to determine which is longer or if they are the same length</td>
</tr>
<tr>
<td>5</td>
<td>Order lengths, marked in 1 to 6 units</td>
</tr>
<tr>
<td>6</td>
<td>Lays units end to end, may not recognize the need for equal length units</td>
</tr>
</tbody>
</table>

Adapted from Clements & Sarama (2009) Learning and Teaching Early Math: The Learning Trajectories Approach

**Killian**

Measurement

- Measuring dinosaurs
Developmentally Appropriate Practice (DAP)

National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM) issued a joint position statement in 2010.

- Identified ten research-based recommendations to guide classroom practice.

Recommended Practices

1. Enhance children’s natural interest in mathematics.
2. Build on experience, knowledge and background.
3. Scaffold teaching practices based on young children’s development.
4. Utilize methods that strengthen problem-solving.
5. Connect mathematical ideas through the curriculum.

National Association for the Education of Young Children and National Council of Teachers of Mathematics (2010)

6. Provide sustained interaction with mathematical ideas.
7. Integrate mathematics with other activities.
8. Provide support for children to engage in play and explore mathematical ideas.
9. Introduce math concepts through experiences and teaching strategies.
10. Assess mathematical knowledge, skills, strategies.

National Association for the Education of Young Children and National Council of Teachers of Mathematics (2010)
Partnering with Parents

- Describe how math concepts are supported in the classroom and encourage parents to practice at home
- Develop take home activities with materials to send home with children
- Include math activities as part of open house or parent-teacher conferences
- NEW math brochure from OSPI for parents and caregivers

Activities from 18-36 month range

- Practice the number sequence (rote counting)
- Point out numbers in their environment
- Have children practice counting small groups of items
- Look for opportunities to use words like more, taller, longer and other comparison words
- Using small groups (1-3) of simple items, ask child, “How many?” In time, they should say without counting

Activities from 36-48 month range

- Extend the sequence of numbers the child knows
- Count larger sets of objects that are in a line
- Look for opportunities to use words like beside and between as well as less than, smaller, and shorter
- Point out numbers in the child’s environment and name them
- Teach the names of basic geometric shapes and find them in the environment
- Ask child to bring you a certain number of items (up to 4)
Activities from 48 month to kindergarten range

- Set up simple situations which illustrate simple addition and subtraction using small numbers
- Set up situations where you can ask if collections of objects have the same number
- Play counting games and memory games with your child
- Teach the names of basic geometric shapes
- Sort items into categories
- Learn what the numbers look like by pointing them out in the environment

Child with a disability


- Three age groups: Birth-2, 3-5, 6-21
- Descriptive study using existing data
- Pre-Elementary Education Longitudinal Study (PEELS)
- Early Childhood Longitudinal Study (ECLS)
Child with a disability

- Numeracy outcomes for preschoolers receiving special education services are significantly lower than those of the general population
  
  Blackorby, et al. (2010)

- The **sequence** of development for children with disabilities is usually commensurate with typically developing peers while the **rate** may be slower
  
  Sadler (2009)

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**Special Education Students Ages 3-21**

<table>
<thead>
<tr>
<th>Disability Category</th>
<th>3</th>
<th>4</th>
<th>5-8</th>
<th>9-14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developmental Delay</td>
<td>214</td>
<td>168</td>
<td>349</td>
<td>867</td>
</tr>
<tr>
<td>Emotional/Behavioral Disability</td>
<td>3</td>
<td>4</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>Orthopedic Impairments</td>
<td>18</td>
<td>24</td>
<td>29</td>
<td>3</td>
</tr>
<tr>
<td>Health Impairments</td>
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<td>146</td>
<td>187</td>
<td>448</td>
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<tr>
<td>Specific Learning Disabilities</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mental Retardation</td>
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<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Multiple Disabilities</td>
<td>22</td>
<td>26</td>
<td>35</td>
<td>78</td>
</tr>
<tr>
<td>Traumatic Brain Injury</td>
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<td>7</td>
<td>21</td>
<td>56</td>
</tr>
<tr>
<td>Hearing Impairments</td>
<td>35</td>
<td>43</td>
<td>48</td>
<td>115</td>
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<tr>
<td>Visual Impairments</td>
<td>24</td>
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<td>30</td>
<td>65</td>
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<tr>
<td>Blindness</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>Communication Disorders</td>
<td>687</td>
<td>1225</td>
<td>1803</td>
<td>3068</td>
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<tr>
<td>Autism</td>
<td>174</td>
<td>204</td>
<td>348</td>
<td>305</td>
</tr>
<tr>
<td>Traumatic Brain Injury</td>
<td>12</td>
<td>4</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

**GRAND TOTALS**: 3222 4095 6038 12275

http://www.k12.wa.us/SpecialEd/Data/Childcount-Placement.aspx

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Child with a disability

- How does the presence of a disability impact early numeracy skills?
  
  - Consider the adverse educational impact of the disability
  
  - Embed mathematical concepts into classroom activities
**Child with a disability**

- Child is almost 3 and has multiple disabilities
- Rote counting to five with mom

**Accommodations and Modifications**

- The IEP team determines what accommodations and/or modifications are necessary for the child
- What accommodations and/or modifications could be used to support the acquisition of early numeracy skills?

**Practice**

- Case Studies
Case Study: Raul

- Raul is a three year old with Down Syndrome. He has IEP goals in the areas of cognitive, communication, gross motor and adaptive skills. Raul’s Head Start teacher is planning an activity next week for the children to count and sort vegetables to make vegetable soup.

- What are some guiding questions Raul’s teacher could ask?

- What accommodations could be made to facilitate Raul’s independent participation in this counting activity?

Case Study: Jasmine

- Jasmine is a four year old with autism. She has IEP goals in the areas of communication, cognition, adaptive and social skills. Her preschool teacher is planning to introduce math concepts into choice time at the sensory table. The sensory table is not always a preferred activity for Jasmine.

- What are some math concepts that could be targeted for Jasmine at the sensory table?

- What accommodations could be made to facilitate Jasmine’s independent participation in sensory table activities?

Case Study: Hunter

- Hunter is a five year old with a speech and language impairment. He has IEP goals in the areas of receptive and expressive language and social skills. One of Hunter’s favorite times of the preschool day is gross motor play.

- How could Hunter’s teacher address math concepts during gross motor activities?

- What accommodations could be made to facilitate Hunter’s independent participation in gross motor activities?
Where to start?

- Encourage play! Young children learn best through active exploration
- Listen to what children say to gauge their current skill level
- Think of the developmental trajectories for counting, comparison, operation, shape and measurement
- Ask guiding questions to increase understanding of math concepts
- Use number words when talking “Let’s go down these 3 steps”, “we’re walking through the two doors”

Summary

1. Reviewed Developmentally Appropriate Practice (DAP)
2. Examined recent research on early numeracy development
3. Introduced early numeracy concept trajectories
4. Illustrated concept trajectories with video examples
5. Identified evidence-based instructional practices to support early numeracy development of all young children, ages three to five
6. Discussed video and case study examples of early math activities with young children who have disabilities

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References


